

Constrained control framework for a stand-alone hybrid (Stirling engine)/supercapacitor power generation system



M. Alamir^{a,*}, M.A. Rahmani^{a,b}, D. Gualino^a

^a CNRS, Gipsa-Lab, Control Systems Department, Grenoble, France

^b Schneider Electric, 37 quai Paul-Louis Merlin, 38050 Grenoble, France

HIGHLIGHTS

- A complete state feedback controller for the voltage conditioning stage of a hybrid power plant is proposed.
- The controller explicitly handles the state and controller constraints.
- The developed control methodology can be applied to various power electronics architectures.

ARTICLE INFO

Article history:

Received 2 July 2013

Received in revised form 28 October 2013

Accepted 18 December 2013

Available online 14 January 2014

Keywords:

Stirling engines

Supercapacitor

Power generation

Voltage regulation

Saturation constraints

Nonlinear control

ABSTRACT

In this paper, a complete control architecture is proposed for the voltage conditioning stage of a hybrid power generation system composed of a Stirling engine coupled with a supercapacitor. Such a solar energy-based generation system aims at providing electricity to off-grid regions. The novelty of the proposed architecture is that it completely handles constraints on all the state variables of the electric stage while providing near to optimal performances in terms of settling time. The derivation of the control law enables a deep understanding of the main issues involved in the success of the closed-loop control. Moreover, the resulting feedback laws are real-time compatible and are given in a complete explicit form.

© 2014 Elsevier Ltd. All rights reserved.

1. Introduction

Today, more than 1.4 billion people have no access to electricity, essentially in India, Indonesia, Bangladesh, Nigeria and Sub-Saharan Africa. Classical alternative fuel-based generators such as Diesel engines suffer from many drawbacks including their low reactivity to load changes, their high emissions and noise to mention but few issues [1].

Stirling engines are based on the Stirling thermodynamic cycle which describes a fluid cycling between a cold and a hot sources in order to transform heat energy into kinetic energy. This means that well isolated tanks containing water that is heated during the sunny hours can be viewed as green batteries that can provide kinetic energy to steer an alternator and produce electricity as soon as required. In addition to this, the Stirling engines have the following advantages: low noise level, robustness, simplicity of the associ-

ated hydraulic circuit and a good thermal to electrical conversion efficiency. Then a micro solar power plant based on a Stirling engine seems to be a promising solution.

However, one of the main issues of such a plant is its low response time when moving from an operating point to another when the supplied electrical load varies. An energy buffer which is a supercapacitor in the studied system is needed to palliate the load power demand in a fast manner while the Stirling engine adapts its produced power in a rather slower manner. Then an efficient control of the Stirling engine through its associated electrical part has a great influence on the dimensioning of the energy buffer that has to be as small as possible to reduce the cost of the plant.

Within this framework, a control architecture for the voltage conditioning stage associated to a Stirling engine was developed and described in this paper. A preliminary solution to this problem has already been proposed in [2] where a one step unconstrained Model Predictive Control has been used. However, the approach proposed in [2] does not explicitly tackle the state constraints. Instead an ad hoc filter has been added in order to enlarge the domain of applicability of the proposed solution.

* Corresponding author.

E-mail addresses: mazen.alamir@gipsa-lab.grenoble-inp.fr (M. Alamir), mustapha.rahmani@schneider-electric.com (M.A. Rahmani), david.gualino@schneider-electric.com (D. Gualino).

Many hybrid power generation systems have been studied in the literature either for grid or off-grid applications. They combine different renewable energy sources essentially wind, solar photovoltaic, hydraulic and fuel cells. Different power electronics architectures associated to those hybrid power systems have been proposed and their corresponding control strategies developed. In [3], a variable speed wind turbine associated to a simple power electronics have been used to supply AC loads, an MPPT (Maximum Power Point Tracking) algorithm was used to extract maximum power from the wind using a DC/DC boost converter while an inverter was used to create the AC grid and to regulate the DC bus, however because of the random nature of the wind, the AC voltage was perturbed. In [4], a similar system combined to a Diesel Engine and a battery bank was studied. The batteries act as a buffer source (through a buck-boost converter) to regulate the DC bus absorbing power in case of high wind and discharging in case of scarce wind. A linearized models were used to control the system at some operating points. However for many applications where the loads and power sources can vary significantly, such controllers are not suitable. In [5], a similar system provided by dump loads and its associated switch was studied and PI based controllers were proposed. In [6] a hybrid wind/photovoltaic/fuel cell power system for grid-connected applications was proposed. An MPPT techniques were used to extract maximum power from the wind and PV panels while the fuel cells system suppress the grid power fluctuations. The objectives of such hybrid systems is to act as a battery-less uninterruptible power source (UPS). Valenciaga et al. ([7–9]) proposed an interesting technique to control a hybrid wind/photovoltaic (and even extra power sources) with wind as a primary source and a battery bank on the DC bus as an energy storage device. The control design proposed by Valenciaga et al. are based on passivity/sliding mode techniques (and a second order sliding mode technique in [9]) for different wind regimes. Supervisory controllers for such systems were proposed in [10–12]. All the techniques previously cited do not handle explicitly the constraints of the system's state variables and controllers which can lead to some undesirable behaviors.

In the present paper, a complete state feedback solution is proposed that entirely solves the constrained problem while providing a deep understanding of the interaction between the different subsystems composing the whole voltage conditioning stage. This is done by decomposing the whole system into subsystems for which the vector fields can be analyzed graphically. A nice feature in the proposed solution is that it suggests a general control design methodology for a family of power circuits despite the fact that its success may depend on the quantitative design of the circuit components. The proposed design underlines the conditions under which the framework is successful and can therefore be also used to guide the design step.

The paper is organized as follows: First of all, the solar micro plant is described in Section 2. The mathematical equations describing the system's dynamics which are used in the design of the control law are described in Section 3. Some preliminary results that are used in the remainder of the paper are derived in Section 4. The explicit expression of the feedback law is described in Section 5 and simulation-based validation is proposed in Section 6 through a wide variety of scenarios in order to strengthen the successful constraint handling capability of the proposed feedback. Finally, Section 7 concludes the paper by summarizing its contributions and by giving hints for future investigations.

2. Description of the solar micro plant

Fig. 1 shows an overview of the solar micro plant including the Stirling engine linked to a synchronous machine together with its voltage conditioning stage that consists of a diode bridge, a full bridge DC/DC converter, a DC bus, a single phase rectifier. A supercapacitor is also present and linked to the DC bus through a bidirectional DC/DC converter.

More precisely, the Stirling engine is a thermodynamic machine that transforms heat (of the hot fluid flowing inside one side of the Stirling engine internal heat exchanger) into a mechanical work by performing a Stirling thermodynamic cycle at each shaft rotation [13]. The produced mechanical torque is used to drive the synchronous machine that reacts by producing an electromagnetic torque which together with the motor torque impose the rotational speed of the shaft. The variable speed of the engine induces a variable amplitude and frequency voltage at the output of the PMSG which is conditioned through the mentioned power electronics stage before feeding the load with a 230 V and 50 Hz voltage. The process of the voltage conditioning is as follow:

The variable amplitude and frequency voltage at the output of the PMSG is rectified through the diode bridge to produce a variable DC voltage which is conditioned through a DC/DC full bridge converter that is connected to a DC bus whose voltage V_{bus} needs to be regulated at the appropriate value of 50 V which is compatible with the good functioning of the inverter that supplies the load with the AC voltage.

Note however that in order to dynamically meet the load demand, the Stirling engine alone is not sufficient due to its high mechanical and thermal inertia. That explains the use of the supercapacitor (see Fig. 1) as a fast energy buffer during highly dynamic transients to tightly regulate the DC bus voltage V_{bus} at 50 V through appropriate management of the control u_2 (duty ratio of the bidirectional DC/DC converter connected to the supercapacitor). The control u_1 (duty ratio of the DC/DC full bridge converter) has then to be used to control the output current of the DC/DC full bridge converter namely I_{Lfb} at some desired reference value

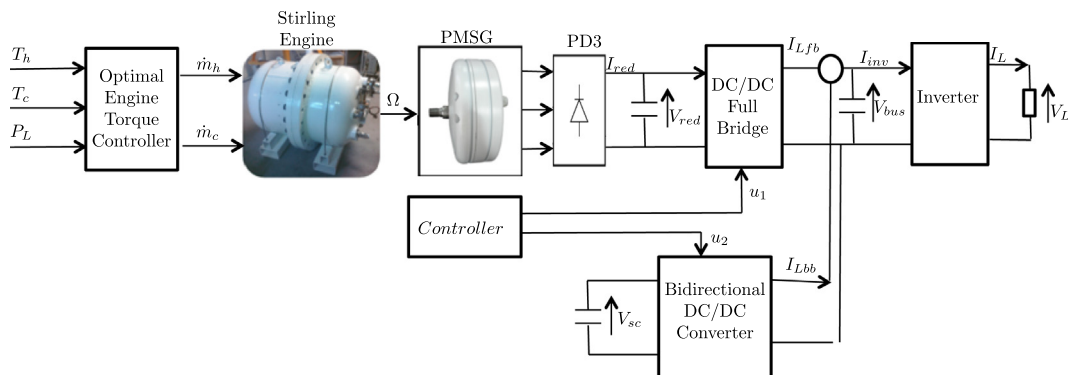


Fig. 1. Overview of the solar micro plant including the Stirling engine together with its voltage conditioning stage.

computed so as to meet the load power demand and to enable the charge/discharge of the supercapacitor in order to recover its desired voltage $V_{sc}^{ref} = 120$ V.

Note that the engine torque controller depicted at the left edge of Fig. 1 is not studied in this contribution. Indeed, this controller uses the information on the power demand, the input temperatures of the hot and cold fluids flowing through the internal heat exchangers of the Stirling engine (see [13]) in order to compute the mass flow rates set-points leading to an optimal thermoelectric efficiency [14]. This loop, while important for the global efficiency of the systems, obviously works in a separated time scale when compared to the characteristic of the voltage conditioning stage. This makes relevant a separated control design of the latter that is based on the assumption of a constant torque at the input of the PMSG block.

3. Mathematical model and problem statement

The Stirling engine studied in this paper is a prototype available within Schneider Electric. It is a 3 kW_{elec} (maximum electrical output at 300 °C for the hot fluid temperature) alpha type Stirling engine (see [13]). From steady state experiments performed on this engine, an expression of the motor torque was fitted to the experimental measurements using the Eureka-Formulize[®] equation fitter ([15,16]) leading to the following result:

$$T_{mot} = \alpha(\dot{m}_h, \dot{m}_c, T_h^{in}, T_c^{in}) \cdot \Omega(t) + \beta(\dot{m}_h, \dot{m}_c, T_h^{in}, T_c^{in}) \quad (1)$$

where T_{mot} is the Stirling engine output torque, $\Omega(t)$ is the shaft speed (in rad/s), $\dot{m}_h$ and $\dot{m}_c$ are the mass flow rates of the hot and cold fluids at the inlet of the Stirling engine, T_h^{in} and T_c^{in} are their respective temperatures. $\alpha(\cdot)$ and $\beta(\cdot)$ are the functions fitted with the experimental measurements and given by:

$$\begin{aligned} \alpha(\dot{m}_h, \dot{m}_c, T_h^{in}, T_c^{in}) &= 0.2498 + 0.004109 \cdot T_h^{in} + 3.13 \times 10^{-5} \cdot T_h^{in} \cdot T_c^{in} \cdot \dot{m}_h \\ &\quad - 1.812 \cdot \dot{m}_h - 1.655 \times 10^{-5} \cdot T_h^{in} \cdot T_c^{in} - 0.007562 \cdot T_c^{in} \cdot \dot{m}_h \\ \beta(\dot{m}_h, \dot{m}_c, T_h^{in}, T_c^{in}) &= 0.1 \cdot T_h^{in} - 30.03 \end{aligned}$$

Fig. 2 compares the Stirling engine torque computed using Eq. (1) with the torque derived from the experimental data for a set of 160 operating points defined by $\dot{m}_h$, $\dot{m}_c$, T_h^{in} and T_c^{in} . It can be seen from Fig. 2 that the curve corresponding to Eq. (1) fits most of the experimental data points.

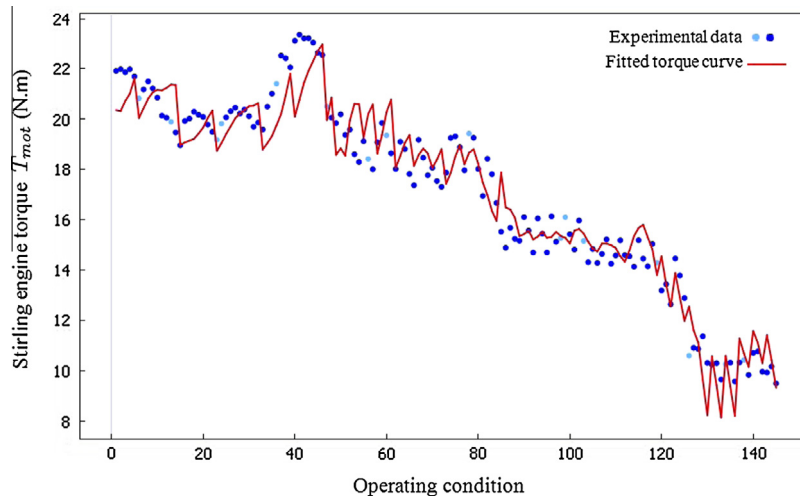


Fig. 2. Comparison between the fitted Stirling engine torque equation and the torque derived from the experimental data. each point in the figure corresponds to an operating condition defined by $\dot{m}_h$, $\dot{m}_c$, T_h^{in} and T_c^{in} .

In a recent contribution [2], it has been shown that the dynamic system lying to the right of the Stirling engine (see Fig. 1) can be described by the following set of ordinary differential equations:

$$\dot{x}_1 = -a_1 x_1 - a_3 x_2 + a_2 \quad (2a)$$

$$\dot{x}_2 = -a_4 x_2 + a_6 x_1 - a_7 x_3 \quad (2b)$$

$$\dot{x}_3 = a_8 (x_2 - k x_4 u_1) \quad (2c)$$

$$\dot{x}_4 = a_9 (-x_5 + k x_3 u_1) \quad (2d)$$

$$\dot{x}_5 = a_{10} \left(x_4 + x_6 - \frac{P_L}{\eta_{inv} \cdot x_5} \right) \quad (2e)$$

$$\dot{x}_6 = a_{11} (-x_5 + x_7 u_2) \quad (2f)$$

$$\dot{x}_7 = -a_{12} x_6 u_2 \quad (2g)$$

where the state components x_i , $i \in \{1, \dots, 7\}$ are associated to the physical variables depicted in Fig. 1 as follows: $x_1 = \Omega$, $x_2 = I_{red}$, $x_3 = V_{red}$, $x_4 = I_{lfb}$, $x_5 = V_{bus}$, $x_6 = I_{lbb}$ and $x_7 = V_{sc}$. The coefficients a_i , $i \in \{1, \dots, 12\}$ are constant parameters depending on the component values according to the expressions given in [2]. η_{inv} involved in (2e) represents an efficiency factor.

Note that the coefficients a_1 and a_2 depend on the mechanical torque of the Stirling engine defined by Eq. (1) and on the equivalent moment of inertia of the rotating shaft (see [2]). According to the comment made at the end of the preceding section, this coefficients are supposed here to be constant as the torque variation is assumed to be very slow when compared to the electric stage characteristic times.

Note also that the Eqs. (2a), (2g), (2c)–(2f) and (2g) describe the mean behavior so that u_1 and u_2 may take all values in $[0, 1]$. The resulting control values are then implemented through standard Pulse Width Modulation (PWM) techniques.

Note finally that the model used for the supercapacitor is simply that of a big capacity whose value being equal to 5F neglecting the equivalent resistance in series since it is generally very small (as it is the case for the real supercapacitor available within Schneider Electric testbed).

As far as numerical values are concerned, the coefficients a_i , $i \in \{1, \dots, 12\}$ for the studied system and for some operating point are given by: $a_1 = -0.183$, $a_2 = 558.11$, $a_3 = 118.4453$, $a_4 = 9615.4$, $a_5 = 1.3712$, $a_6 = 5101.1$, $a_7 = 641.02$, $a_8 = 425.53$, $a_9 = 6250$, $a_{10} = 7.34$, $a_{11} = 4484.3$, $a_{12} = 0.2$, $\eta_{inv} = 0.95$ and $k = 1$.

Rephrasing the control objective in terms of the new state variables leads to a control objective aiming at regulating x_5 and x_7 around their desired values which are $x_5^r = 50$ V and $x_7^r = 120$ V

respectively. Moreover, the validity of the model together with the electrical requirement imposes the following set of constraints:

$$x_i \in [x_i^{\min}, x_i^{\max}]; \quad i \in \{2, 3, 4, 6\} \quad (3)$$

The constraints on the regulated variables x_5 and x_7 will be naturally respected by the success of the control loop while the controlled excursion on x_1 is implicitly imposed through the constraints on both x_2 and x_3 as it can be inferred from Eqs. (2a) and (2b).

4. Preliminary results

In this section some preliminary results that are used in the sequel are stated and, when needed, proved. Let us first begin by the following assumptions that are checked to be satisfied for the studied system:

Assumption 1. The linear system in the state (x_1, x_2) defined by (2a), (2b) (steered by the exogenous signal x_3) shows two separated negative real eigenvalues λ_f and λ_s such that $|\lambda_f| \gg |\lambda_s|$.

For the studied system, this assumption is satisfied with $|\lambda_f| \approx 9552 \text{ Hz} \gg |\lambda_s| \approx 63.4 \text{ Hz}$. This feature is likely to be satisfied on any testbed as it reflects the co-existence of mechanical (x_1) and electrical (x_2) variables which generally evolve among separated time scales.

Assumption 2. The bounds $x_3^{\min}$ and $x_3^{\max}$ invoked in (3) are compatible with the constraints on x_2 , namely the steady values of x_2 belong to $[x_2^{\min}, x_2^{\max}]$ for all constant value of $x_3 \in [x_3^{\min}, x_3^{\max}]$. Moreover, $x_3^{\min} > x_5^r$.

This assumption can be checked on Fig. 3 (see also Assumption 3 below) where all possible steady states of (2a)–(2c) and (2d) in which $x_5 = x_5^r$ is used, are depicted as functions of u_1 . It can be easily checked that a wide variation range of x_3 corresponds to a small variation range of x_2 . In particular, Assumption 2 is clearly satisfied as soon as the interval $[x_2^{\min}, x_2^{\max}]$ contains the interval $[4.736, 4.919]$. Note that the need for the condition $x_3^{\min} > x_5^r$ can be easily understood by observing Eq. (2d) which indicates that the controllability of x_4 using $u_1 \in [0, 1]$ (when $x_5 \approx x_5^r$) requires the inequality $x_3 > x_5^r$ to be satisfied.

The first result states that it is possible to monitor the fulfillment of the constraints on the rectified current $x_2 = I_{red}$ by

monitoring the excursion of the rectified voltage $x_3 = V_{red}$. More precisely, the following lemma is derived:

Lemma 1. There are easily computable maps $\underline{x}_3(x_1, x_2)$ and $\bar{x}_3(x_1, x_2)$ such that all trajectories of (2a), (2b) starting from an admissible initial conditions (regarding the constraints on x_2) and steered by a profile $x_3(\cdot)$ meeting the following condition:

$$x_3(t) \in [\underline{x}_3(x_1(t), x_2(t)), \bar{x}_3(x_1(t), x_2(t))] \quad (4)$$

satisfy the saturation constraints on x_2 .

Proof. See appendix A where explicit expressions (A.2), (A.3) are given for $\underline{x}_3(x_1, x_2)$ and $\bar{x}_3(x_1, x_2)$. $\square$

The next assumption concerns the dynamic of the subsystem described by Eqs. (2a)–(2c) and (2d) in which the state is given by $z := (x_1, x_2, x_3, x_4)^T \in \mathbb{R}^4$ while x_5 is assumed to be equal to its reference value, namely: $x_5 \equiv x_5^r = 50V$. Such a consideration is relevant as the tight regulation of x_5 around x_5^r will be achieved using the control u_2 as it is shown later. Obviously, under these conditions, the system (2a)–(2c) and (2d) can be written in the following condensed form:

$$\dot{z} = A(u_1)z + B \begin{pmatrix} a_2 \\ x_5^r \end{pmatrix} \quad (5)$$

Assumption 3. For each positive value $u_1 \in [0, 1]$, the matrix $A(u_1)$ is Hurwitz. Consequently, for each such constant value of u_1 , the system (5) shows steady states that are denoted hereafter by $x_i^{st}(u_1)$, $i \in \{1, \dots, 4\}$.

The evolution of the steady state values $x_i^{st}(u_1)$, $i \in \{1, \dots, 4\}$ are depicted in Fig. 3. This figure shows a very interesting feature regarding the evolution of $x_2^{st}(u_1)$. Indeed, it is remarkable that a wide range of values of x_4 can be asymptotically produced (which is crucial in the sequel when it comes to regulate the super-capacitor voltage $x_7 = V_{sc}$) for a rather very restricted range of values of x_2 (between 4.736 A and 4.919 A). In order to refer to this feature in the forthcoming argumentation, the following definition is used:

Definition 1. The range of variations of steady values of x_2 is denoted hereafter by ε_2 , namely

$$\varepsilon_2 := \max_{u_1 \in [0, 1]} x_2^{st}(u_1) - \min_{u_1 \in [0, 1]} x_2^{st}(u_1) \quad (6)$$

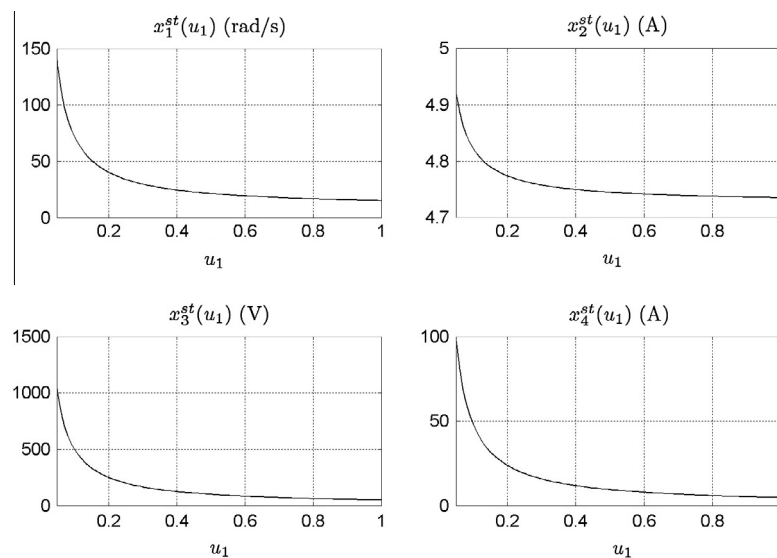


Fig. 3. Illustration of Assumption 3: Evolution of the steady state values $x_i^{st}(u_1)$, $i \in \{1, \dots, 4\}$ defined in Assumption 3 as a function of u_1 for the studied system.

while the central value of $x_2^{st}(\cdot)$ is denoted hereafter by $x_2^{st,c}$. Consequently, $x_2^{st}(u_1) \in \mathcal{I}_2 := x_2^{st,c} + [-\frac{\varepsilon_2}{2}, +\frac{\varepsilon_2}{2}]$.

Note that the parameters ε_2 and $x_2^{st,c}$ are determined by the dimensioning of the system and does not depends on any control design.

Fig. 3 shows that for the studied system, $x_2^{st,c} \approx 4.82$ while $\varepsilon_2 \approx 0.12$ which corresponds to a relative variation of less than 2.5%.

Another feature that can be observed in Fig. 3 is that all the maps are monotonic so that the following notation can be used:

$$x_i^{st}(x_j^{st}), u_1^{st}(x_j^{st}) \quad (i, j) \in \{1, \dots, 4\}$$

to denote the stationary values of x_i and u_1 that correspond to a given stationary value x_j of the j th component of the state vector z . This is extensively used in the sequel.

Based on the observation made earlier regarding the excursion of the stationary values of x_2 , it can be argued that if the conclusion of Lemma 1 is used to define bounds on the variations of x_3 such that x_2 remains in a tight interval around $x_2^{st,c}$ (see Definition 1), then it becomes relevant to analyze the behavior of the dynamic system (2c) and (2d) in which the state is (x_3, x_4) while x_2 and x_5 are considered to be constant. Such an analysis can then be performed by visualizing the vector fields in the plan (x_3, x_4) . When doing this analysis using the numerical values mentioned at the end of Section 3, it comes that the following assumption is satisfied:

Assumption 4. For any admissible desired pair $(x_3^*, x_4^*(x_3^*))$, the picture of the vector fields of the dynamic system (2c)–(2d) in which $x_2 = x_2^{st}(x_3^*) \in \mathcal{I}_2$ and $x_5 = x_5^*$ is the one depicted in Fig. 4 below, more precisely:

1. The horizontal lines AB and DE correspond to the lines $x_4 = x_4^{min}$ and $x_4 = x_4^{max}$ respectively.
2. C is the desired steady point $(x_3^*, x_4^*(x_3^*))$ while $u_1^* = u_1^{st}(x_3^*)$ is the corresponding steady control.
3. DC is the trajectory under the control $u_1 = 0$ that starts at D and ends at C (this defines D).
4. BC is the trajectory under the control $u_1 = 1$ that starts at B and ends at C (this defines B).
5. The disposition of the vector fields on the line AB and DE are such that the control $\bar{u}_1(x)$ given by:

$$\bar{u}_1(x) := \begin{cases} \frac{1}{kx_3} [x_5 + \rho_4(x_4^{max} - x_4)] & \text{on DE} \\ \frac{1}{kx_3} [x_5 + \rho_4(x_4^{min} - x_4)] & \text{on AB} \end{cases} \quad (7)$$

steers the state towards D (resp. B) when $x_4 = x_4^{max}$ (resp. x_4^{min}) while keeping x_4 constant.

6. The line ABCDE is attractive under the control $u_1 = 1$ for any starting point that lies to the right of the curve BCD (this region is denoted by $\mathcal{R}_{34}$ on Fig. 4).
7. The line ABCDE is attractive under the control $u_1 = 0$ for any starting point that lies to the left of the curve BCD (this region is denoted by $\mathcal{L}_{34}$ on Fig. 4)

Note that the first 4 items of Assumption 4 are only definitions. The item (5) can be easily checked by investigating the disposition of the vector fields on the horizontal lines AB and DE. It is also possible to formulate algebraic conditions [see B.1, B.2, B.3 and B.4] to check items (5), (6) and (7) above as it is shown in appendix B.

Although the conditions B.1, B.2, B.3 and B.4 might seem tricky to check, it is likely that the topology of the vector fields depicted in Fig. 4 holds for a wide class of such voltage conditioning systems. The above conditions are only given for completeness.

Note also that the disposition of the vector fields obviously suggests that a sliding modes controller can be defined in which the

curves ABCDE is the sliding surface while the equivalent control on the sliding surface steers the system towards the desired point C. Here we prefer to use continuous control because the Eqs. (2a)–(2c) and (2d) represent the mean dynamic and in order to derive an easy to use closed-loop systems avoiding the numerical difficulties associated to the sliding modes controllers simulation when long tuning and assessing scenarios are required as it is the case here (see later).

Rather than using the sliding modes formulation, a 2D interpolation techniques is used in the (x_3, x_4) plan. Namely, an interpolated expression of the feedback is derived in which, the expressions $u_1 = 1$ or $u_1 = 0$ are obtained when the current state (x_3, x_4) is far from the curve ABCDE while u_1 takes the appropriate values when (x_3, x_4) lies on the curve ABCDE. The appropriate value is itself obtained by appropriate interpolation between the values depicted on Fig. 4 according to the position of the current point (x_3, x_4) on the curve ABCDE.

In order to properly define the 2D interpolated expression, the following auxiliary function is needed:

$$\varphi(r, v_1, v_2) := v_1 + \text{Sat}(1 - e^{-\lambda r}) \cdot (v_2 - v_1) \quad (8)$$

which simply takes the value v_1 for $r = 0$ and v_2 when $r \gg 0$ with a transition rate defined by the positive constant λ .

Based on the above discussion, the following explicit feedback can be defined for a given target point $(x_3^*, x_4^*(x_3^*))$:

Definition 2. Let a desired point $(x_3^*, x_4^*(x_3^*))$ be defined. Given a current position $p := (x_3, x_4)$ such that $x_4 \in [x_4^{min}, x_4^{max}]$, compute the following quantities:

- Compute $u_1^{DE}(x)$ and $u_1^{AB}(x)$ according to:

$$u_1^{DE}(x) := \varphi\left(\frac{x_3 - x_3(D)}{x_3(D)}, 0, \bar{u}_1(x)\right) \quad (9)$$

$$u_1^{AB}(x) := \varphi\left(\frac{x_3(B) - x_3}{x_3(B)}, 1, \bar{u}_1(x)\right) \quad (10)$$

where $\bar{u}_1(x)$ is given by (7).

- Compute $u_1^{BCD}(x)$ according to:

$$u_1^{BCD}(x) := \begin{cases} \varphi\left(\frac{x_4^{max} - x_4}{x_4^{max} - x_4^{min}}, 0, u_1^*\right) & \text{if } x_4 > x_4^* \\ \varphi\left(\frac{x_4 - x_4^{min}}{x_4^{max} - x_4^{min}}, 1, u_1^*\right) & \text{if } x_4 \leq x_4^* \end{cases}$$

where $u_1^* := u_1^{st}(x_3^*)$.

- Compute the abscissa $\hat{x}_3$ of the horizontal projection of p on BCD.

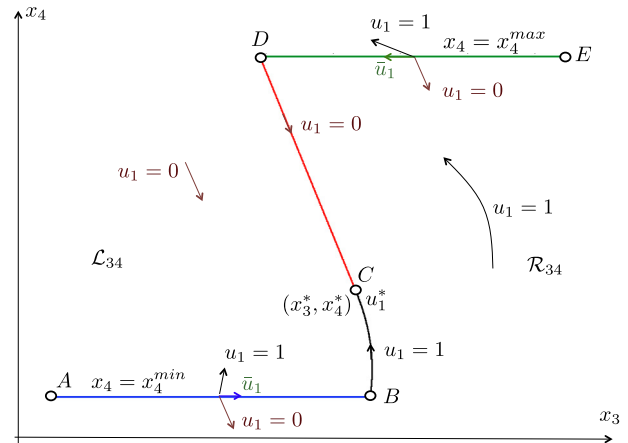


Fig. 4. Typical disposition of the vector fields of the dynamic system (1c) and (1d) when $x_2 = x_2^{st}(x_3)$ and $x_5 = x_5^*$ are assumed to be constant.

- Compute the vertical interpolation $u_1^v(x)$ according to:

$$u_1^v(x) := \begin{cases} \varphi\left(\frac{x_4^{\max} - x_4}{x_4^{\max} - x_4^{\min}}, u_1^{DE}(x), 1\right) & \text{if } x_3 > \hat{x}_3 \\ \varphi\left(\frac{x_4 - x_4^{\min}}{x_4^{\max} - x_4^{\min}}, u_1^{AB}(x), 0\right) & \text{if } x_3 \leq \hat{x}_3 \end{cases}$$

- Compute $u_1(x | x_3^*)$ by horizontal interpolation between $u_1^v(x)$ and $u_1^{BCD}(x)$ according to:

$$u_1(x | x_3^*) := \varphi\left(\frac{|x_3 - \hat{x}_3|}{|\hat{x}_3|}, u_1^{BCD}(x), u_1^v(x)\right) \quad (11)$$

Note that the resulting feedback control $u_1(x | x_3^*)$ defined by (11) is continuous with respect to x for a given target $(x_3^*, x_4^*) = (x_3^*, x_4^{st}(x_3^*))$. Indeed, for a given value of x_4 , continuity w.r.t x_3 is guaranteed by the continuity w.r.t x_3 of the expressions (9), (10) and (11) while the continuity w.r.t x_4 for a given x_3 is guaranteed by the continuity w.r.t x_4 of the expressions of $u_1^{BCD}(x)$ and $u_1^v(x)$.

Note that the success of the state feedback $u_1(x | x_3^*)$ in steering the state (x_3, x_4) to the desired (x_3^*, x_4^*) is conditioned by the fact that the excursion of x_2 is small. Indeed, under this assumption, the picture depicted in Fig. 4 remains relevant while x_2 is slightly moving. Note however that for this excursion to be small, the following two requirements are needed:

- The range of variation ε_2 of the steady values of x_2 (invoked in Definition 1) is small.
- The variations of x_3 remains inside the moving interval $[\underline{x}_3(x_1, x_2), \bar{x}_3(x_1, x_2)]$ which is computed online based on the admissible range $[x_2^{\min}, x_2^{\max}]$ [see (A.2), (A.3) in the appendix].

The first requirement mentioned above depends on the choice of the system's components and has to be satisfied by construction. The second requirement can be satisfied by an appropriate management of the desired value x_3^* . This is explained in the following discussion.

First of all, the following lemma is needed:

Lemma 2. Any appropriate control that asymptotically steers x_5 to its reference value x_5^r leads to the following property:

$$\lim_{t \rightarrow \infty} [x_6(t) - x_6^r(t)] = 0 \quad (12)$$

where $x_6^r(t)$ is given by:

$$x_6^r(t) = \frac{P_L}{\eta_{inv} \cdot x_5^r} - x_4(t) \quad (13)$$

where P_L is the load power demand.

Proof. This directly follows from Eq. (2e) in which x_5 is replaced by x_5^r while $\dot{x}_5 = 0$ is used. $\square$

It comes from Lemma 2 together with Eq. (2g) that it is possible to regulate the supercapacitor voltage x_7 by choosing the following reference value for x_4 :

$$x_4^r(x, P_L) := \frac{P_L}{\eta_{inv} \cdot x_5^r} + \delta_7 \cdot \text{Sign}[x_7^r - x_7]; \quad \delta_7 > 0 \quad (14)$$

since according to (12), (13) this asymptotically infers the following dynamics to x_7 :

$$\dot{x}_7 = a_{12} \cdot u_2 \cdot \delta_7 \cdot \text{Sign}(x_7^r - x_7) \quad (15)$$

which obviously stabilizes x_7 around x_7^r .

Note however that for (15) to be compatible with the constraints on x_4 , the following conditions are needed:

$$x_4^{\min} + \delta_7 \leq \frac{P_L}{\eta_{inv} \cdot x_5^r} \leq x_4^{\max} - \delta_7 \quad (16)$$

This obviously introduces constraints on the possible load power levels that are compatible with a sustainable operation. This is rigorously stated in the following assumption:

Assumption 5. There is a positive real δ_7 such that all possible values P_L of the load power demand satisfy the inequalities (16).

Before the control law $u_2 = K_2(x | P_L)$ can be completely defined, the following lemma is needed:

Lemma 3. The state feedback law given by:

$$u_2^s(x) := \frac{1}{a_{11} \cdot x_7} \cdot (a_{11} \cdot x_5 - a_{10} \cdot (x_5 - x_5^r) + \dot{x}_6^{\text{ref}} - \rho_6 \cdot (x_6 - x_6^{\text{ref}})) \quad (17)$$

where:

$$x_6^{\text{ref}} = \frac{P_L}{\eta_{inv} \cdot x_5} - x_4 + \frac{\rho_5}{a_{10}} (x_5^r - x_5) \quad (18)$$

in which ρ_5 and ρ_6 are positive design parameters, locally asymptotically steers the pair $(x_5(t), x_6(t))$ to the reference $(x_5^r, x_6^r(t))$, namely:

$$\lim_{t \rightarrow \infty} \left\| \begin{pmatrix} x_5(t) \\ x_6(t) \end{pmatrix} - \begin{pmatrix} x_5^r \\ x_6^r(t) \end{pmatrix} \right\| = 0 \quad (19)$$

Proof. See Appendix C. $\square$

Now we have all we need to define the reference value x_3^* that is used in Definition 2 to compute the state feedback $u_1(x | x_3^*)$. This is defined to be the closest value to $x_3^r(x_4^r)$ [see (14)] that is compatible with the requirement on the excursion of x_2 . More precisely:

$$x_3^*(x, P_L) := \arg \min_{z_3 \in [\underline{x}_3(x), \bar{x}_3(x)]} |z_3 - x_3^r(x_4^r(x, P_L))| \quad (20)$$

where $x_4^r(x, P_L)$ is given by (14) depending on the current value of the supercapacitor voltage x_7 and the present load demand P_L . Note that by doing so, the rate with which the supercapacitor voltage is regulated around its reference value is moderated by the dynamic of the mechanical part of the system which is directly linked to the Stirling engine performance. This is done through the definition of the admissible interval $[\underline{x}_3(x_1(t), x_2(t)), \bar{x}_3(x_1(t), x_2(t))]$.

Remark 1. Note that in [2], it has been observed that in order to avoid negative excursions on some of the system's variables, the variation of the set-point x_4^r needs to be filtered or slowed down. However, contrary to [2] where this feature has been observed, the discussion above gives a deeper interpretation and a more systematic, efficient and dynamic way to address this limitation. Indeed, while the filter introduced in [2] operates continuously slowing down the system unconditionally, the values that result from (20) slow down the system only when necessary. This is a general fact making explicit handling of the constraint preferable to worst-case-based unconstrained design.

The preceding discussion completely defines the feedback law $u_1 = K_1(x, P_L) := u_1(x | x_3^*(x, P_L))$

The remaining part of this section is dedicated to the definition of the feedback law $u_2 = K_2(x, P_L)$. Fortunately, the same graphical analysis can be done in the 2D plan (x_5, x_6) for a given value of x_4 [see Eqs. 2e, 2f]. This analysis is relevant under (20) since as it is explained earlier, Eq. (20) leads to rather slow variations of (x_3, x_4) inducing hence two separated time scales, a slow one for

x_i , $i \in \{1, 2, 3, 4, 7\}$ on one hand and a fast one for x_5 and x_6 on the other hand.

When considering the vector fields of (2e)–(2f) for a given value of x_4 , it can be shown that the following assumption holds for the studied system:

Assumption 6. For a given admissible x_4 and a supercapacitor voltage $x_7 > x_5^r$, the disposition of the vector fields around the desired pair (x_5^r, x_6^r) where x_6^r is defined by (13) is the one given in Fig. 5 below: more precisely:

1. The horizontal lines AB and DE correspond to the lines $x_6 = x_6^{min}$ and $x_6 = x_6^{max}$ respectively.
2. C is the desired steady point (x_5^r, x_6^r) while u_2^* is the locally stabilizing feedback invoked in Lemma 3:
3. DC is the trajectory under the control $u_2 = 0$ that starts at D and ends at C (this defines D).
4. BC is the trajectory under the control $u_2 = 1$ that starts at B and ends at C (this defines B).
5. The disposition of the vector fields on the line AB and DE are such that the controls $\bar{u}_2$ given by:

$$\bar{u}_2(x) := \begin{cases} \frac{1}{x_7} \left[x_5 + \frac{p_6}{a_{11}} (x_6^{max} - x_6) \right] & \text{on DE} \\ \frac{1}{x_7} \left[x_5 + \frac{p_6}{a_{11}} (x_6^{min} - x_6) \right] & \text{on AB} \end{cases} \quad (21)$$

steers the state towards D (resp. B) when $x_6 = x_6^{max}$ (resp. when $x_6 = x_6^{min}$) while keeping x_6 constant.

6. The line ABCDE is attractive under the control $u_2 = 1$ for any starting point that lies to the left of the curve BCD (this region is denoted by $\mathcal{L}_{56}$ on Fig. 5).
7. The line ABCDE is attractive under the control $u_2 = 0$ for any starting point that lies to the right of the curve BCD (this region is denoted by $\mathcal{R}_{56}$ on Fig. 5).

Note that the same comments that followed Assumption 4 can be reproduced here regarding the items invoked in Assumption 6. In particular, there is here also the possibility to check them through explicit conditions similar to (B.1)–(B.3) and (B.4).

Based on the observation of Fig. 5, an interpolation-based state feedback $K_2(x | P_L)$ can be defined in a similar way to the one already used in Definition 2 to define $K_1(x | P_L)$.

Definition 3. Let be given a load power demand P_L . Consider the corresponding desired pair (x_5^r, x_6^r) using x_6^r defined by (13). Compute the following quantities based on the current point $p = (x_5, x_6)^T$:

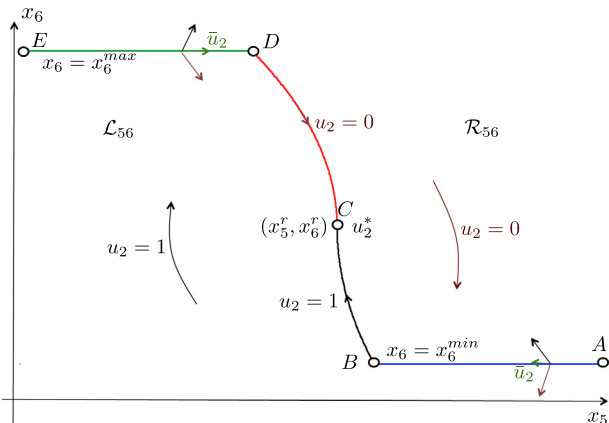


Fig. 5. Typical disposition of the vector fields of the dynamic system (1e) and (1f) for a given x_4 and for $x_7 > x_5^r$.

- Compute $u_2^{DE}(x)$ and $u_1^{AB}(x)$ according to:

$$u_2^{DE}(x) := \varphi \left(\frac{x_5(D) - x_5}{x_5(D)}, 0, \bar{u}_2(x) \right) \quad (22)$$

$$u_2^{AB}(x) := \varphi \left(\frac{x_5 - x_5(B)}{x_5(B)}, 1, \bar{u}_2(x) \right) \quad (23)$$

where $\bar{u}_2(x)$ is given by (21).

- Compute $u_2^{BCD}(x)$ according to:

$$u_2^{BCD}(x) := \begin{cases} \varphi \left(\frac{x_6^{max} - x_6}{x_6^{max}}, 0, u_2^*(x) \right) & \text{if } x_6 > x_6^r \\ \varphi \left(\frac{x_6 - x_6^{min}}{|x_6^{min}|}, 1, u_2^*(x) \right) & \text{if } x_6 \leq x_6^r \end{cases}$$

where $u_2^*(x)$ is given by (17).

- Compute the abscissa $\hat{x}_5$ of the horizontal projection of p on BCD
- Compute the vertical interpolation $u_2^v(x)$ according to

$$u_2^v(x) := \begin{cases} \varphi \left(\frac{x_6^{max} - x_6}{x_6^{max}}, u_2^{DE}(x), 1 \right) & \text{if } x_5 < \hat{x}_5 \\ \varphi \left(\frac{x_6 - x_6^{min}}{|x_6^{min}|}, u_2^{AB}(x), 0 \right) & \text{if } x_5 \geq \hat{x}_5 \end{cases}$$

- The feedback $u_2 = K_2(x | P_L)$ is obtained by horizontal interpolation between $u_2^v(x)$ and $u_2^{BCD}(x)$ according to:

$$K_2(x | P_L) := \varphi \left(\frac{|x_5 - \hat{x}_5|}{|\hat{x}_5|}, u_2^{BCD}(x), u_2^v(x) \right) \quad (24)$$

5. The state feedback law

Note that by definition of the interpolated feedback laws $K_1(x, P_L)$ and $K_2(x, P_L)$, the resulting closed-loop trajectories passes through neighborhoods $\mathcal{V}_{34}(x_3^*, x_4^*)$ and $\mathcal{V}_{56}(x_5^r, x_6^r)$ of the targeted pairs (x_3^*, x_4^*) and (x_5^r, x_6^r) of size $O(1/\lambda)$ after a finite time. In other words by choosing $\lambda > 0$ (used in the interpolation map (8)) sufficiently high, the trajectories cross neighborhoods that are as small as required. In particular, $\lambda > 0$ can be taken sufficiently high so as to make these neighborhoods entirely contained in the basins of attraction of the locally asymptotically stable dynamics under the local control laws $u_1^{st}(x_3^*)$ and $u_2^*(x)$. If then, the feedbacks $u_1^{st}(x_3^*)$ and $u_2^*(x)$ are maintained, then asymptotic stability follows.

The preceding discussion suggests a dual-mode controller. Namely, when the state (x_3, x_4) [resp. (x_5, x_6)] is far from the desired pair, the interpolated law K_1 [resp. K_2] is used to steer the state to small neighborhood of the desired pair. When the state is inside this neighborhood, a switch to the locally asymptotically stabilizing law $u_1^{st}(x_3^*)$ [resp. $u_2^*(x)$] is operated.

It is worth underlying also that if the neighborhoods are taken sufficiently small (that is λ is taken sufficiently high), then the transient excursions still meet all the saturation constraints on the states. The overall control law is therefore admissible.

Note however that such a dual-mode control design has to be done with care in order to avoid undesirable chattering-like behavior (the state enters and leaves the small neighborhood with a high frequency). This is generally avoided by defining the size of the final neighborhoods using the Lyapunov functions level of the local feedback. This is more rigorously defined as follows:

Definition 4. For each admissible x_3^* , the Lyapunov matrix reflecting the stability of $A(u_1^{st}(x_3^*))$ is denoted by $S_{34}(x_3^*)$. Moreover, for any given pair (x_3, x_4) , the following function is defined:

$$d_{34}(x, P_L) := \left\| \begin{pmatrix} 0 \\ 0 \\ x_3 - x_3^* \\ x_4 - x_4^{st}(x_3^*) \end{pmatrix} \right\|_{S_{34}(x_3^*)} \quad (25)$$

where x_3^* is given by (20).

Definition 5. Define $S_{56} \in \mathbb{R}^2$ to be the Lyapunov matrix corresponding to the following Hurwitz matrix:

$$\begin{pmatrix} -\frac{a_{10}P_L}{\eta_{inv}(x_5^r)^2} & a_{10} \\ 0 & -\rho_6 \end{pmatrix} \quad (26)$$

Moreover, for any pair (x_5, x_6) , the following function can be defined:

$$d_{56}(x, P_L) := \left\| \begin{pmatrix} x_5 - x_5^r \\ x_6 - x_6^r(x, P_L) \end{pmatrix} \right\|_{S_{56}} \quad (27)$$

where x_6^r is defined by (13).

By now, we have everything we need to define the complete state feedback:

Definition 6. Let $\lambda > 0$ be the positive scalar invoked in the definition of the interpolation map (8). Let $\gamma_{34} > 0$ and $\gamma_{56} > 0$ be some positive reals. Let P_L be a load power demand. The state feedback law is defined as follows:

$$u_1 := \begin{cases} K_1(x, P_L) & \text{if } d_{34}(x, P_L) > \gamma_{34} \\ u_1^{st}(x_3^*(x, P_L)) & \text{otherwise} \end{cases} \quad (28)$$

$$u_2 := \begin{cases} K_2(x, P_L) & \text{if } d_{56}(x, P_L) > \gamma_{56} \\ u_2^*(x, P_L) & \text{otherwise} \end{cases} \quad (29)$$

Based on the discussions above, the following result can be proved:

Proposition 1. If Assumptions 1–6 are satisfied, then there exist sufficiently small $\gamma_{34} > 0$, $\gamma_{56} > 0$ and sufficiently high $\lambda \times \max(\gamma_{34}, \gamma_{56}) > 0$ such that the closed-system obtained by applying the state feedback laws (28), (28) to the dynamic system (2a)–(2f) and

(2g) asymptotically stabilizes the system's state at a steady value that is compatible with the desired values x_5^r and x_7^r of x_5 and x_7 respectively while meeting the problem's constraints (3) and using admissible control $u \in [0, 1]^2$.

Proof. See Appendix D. $\square$

6. Simulation-based validation

In this section some simulations are proposed to illustrate several facts regarding the proposed state feedback design. More precisely, the validation is split into two stages:

1. In the first stage, referred hereafter as the *decoupled simulations*, each feedback law is validated separately by decoupling the two subsystems. This is done by simulating:
 - the system with state $(x_1, x_2, x_3, x_4)^T$ in which $x_5 = x_5^d$ is used,
 - the system with state (x_5, x_6, x_7) with a given and constant value of x_4 .

These decoupled simulations enable us to show the impact of the saturation on the different state variables on the settling time of the corresponding controlled system to be shown. This stage obviously shows that there are two separate time scales which explains the success of the whole scheme.

2. In the second stage, the whole closed-loop system is precisely simulated under varying power demand in order to assess the ability of the control law to tightly regulate the bus voltage x_5 under varying power demand P_L while regulating in a slower time scale the super-capacitor voltage x_7 .

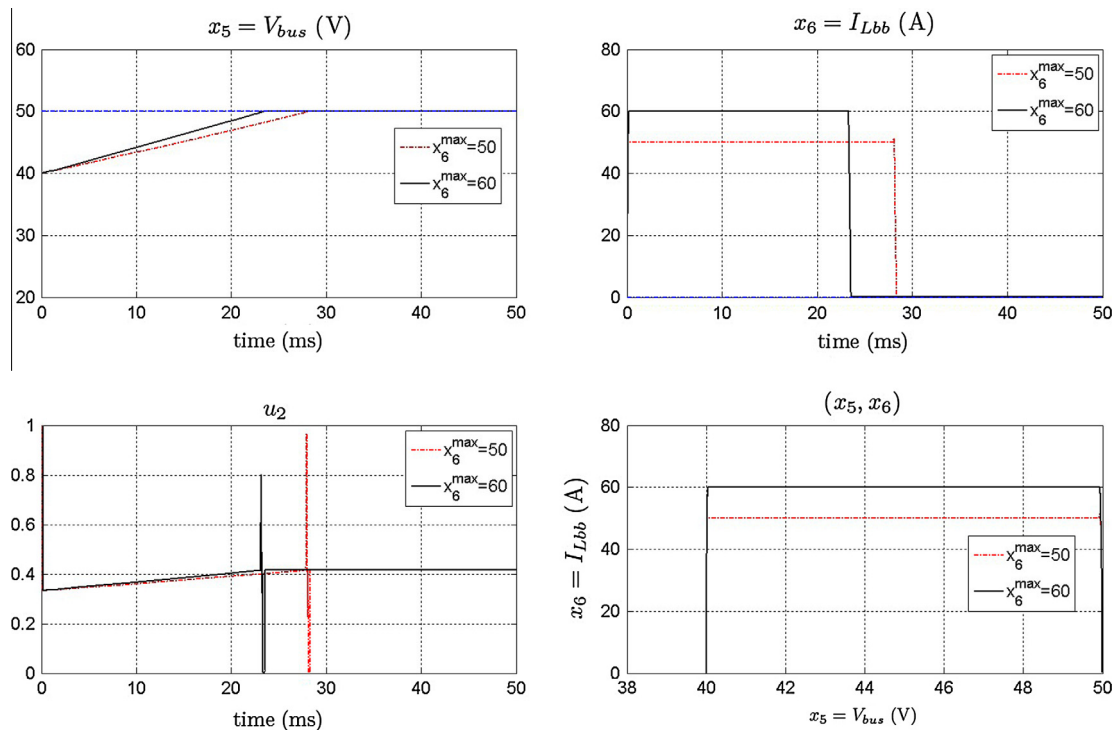


Fig. 6. Control of the (x_5, x_6) system under constant x_4 . Influence of the allowable values of x_6 on the settling time. Comparison between the cases $x_6^{\max} = 50$ and $x_6^{\max} = 60$. Note that the time axis is in milliseconds. The closed-loop system shows a minimum-time like behavior since it saturates the constraints on the current $x_6 = I_{Lbb}$ in order to accelerate the convergence.

6.1. Decoupled simulations

Fig. 6 shows how the voltage x_5 is regulated around the desired value $x_5^d = 50$ V while the desired value $x_4^d = 14.7$ is considered. Two scenarios are plotted in this Figure for two different values of the maximum current $x_6^{max} \in \{50, 60\}$. These scenarios clearly shows that closed-loop behavior shows a sort of minimum-time behavior since it saturates the current in order to accelerate the convergence towards the desired value. Fig. 7 shows how the state x_4 (and therefore x_3) are steered to the desired steady pair (x_3^d, x_4^d) which corresponds to +20% increase in the value of x_4^d while meeting the constraints on all the state variable. In particular, a minimum value of $x_2^{min} = 4$ A. This figures clearly shows how the bounding values defined by (A.2), (A.3) which guarantee the fulfillment of the constraints on x_2 slow down the possible evolution of the pair (x_3, x_4) . Fig. 8 shows the same kind of results when a decrease of the desired value x_4^d of -20%. Note that in this case, the upper bound $x_2^{max} = 4.5$ A is saturated in order to accelerate the convergence. It is worth noting that during the scenarios of Figs. 7 and 8, the state variable x_4 itself does hit the constraints during the first milliseconds. This is difficult to see because of the time scale. This is the reason why the same evolutions are plotted in Figs. 11 and 12 using a logarithmic time scale.

Figs. 9 and 10, shows the impact of the saturation level x_2^{min} and x_2^{max} on the state variable x_2 on the response time of the closed-loop system when $\pm 20\%$ change in the desired value x_4^d is applied. Here again, the closed-loop shows a sort of minimum time behavior since it systematically saturates the constraints in order to accelerate the convergence.

6.2. Global simulation

The decoupled set of simulations proposed in Section 6.1 clearly shows that the time scale of the two subsystems are quite separated. Indeed, the response time of the bus voltage related subsystem is in the range of milliseconds while the response time of the current $x_4 = I_{Lfb}$ that is used to charge the super-capacitor voltage is in the range of seconds. This justifies the decoupled design. Nevertheless, the global control design is validated in the present section through closed-loop simulation of the whole system under varying power demand scenario.

Figs. 13–15 show three different scenarios that investigate the overall closed-loop performance under time varying power load demand profiles. In particular, Figs. 13 and 14 show two scenarios that differ only by the maximal allowed value x_2^{max} on the current $x_2 = I_{red}$ which is set to $x_2^{max} = 5$ A and $x_2^{max} = 4.8$ A respectively. Note how this slight difference has a great impact on the amplitude of the excursion of the supercapacitor voltage $x_7 = V_{sc}$. This suggests that the maximum allowable current x_2 which heavily depends on the choice of the component has to be jointly designed with the choice of the supercapacitor.

All the simulations clearly show that the control objectives are met in the sense that the bus voltage is tightly regulated while the supercapacitor voltage is restored as soon as the constraints on the different control and state variables enable it.

7. Conclusion

In this paper, a global constrained control framework is proposed for the stand-alone hybrid (stirling engine)/supercapacitor

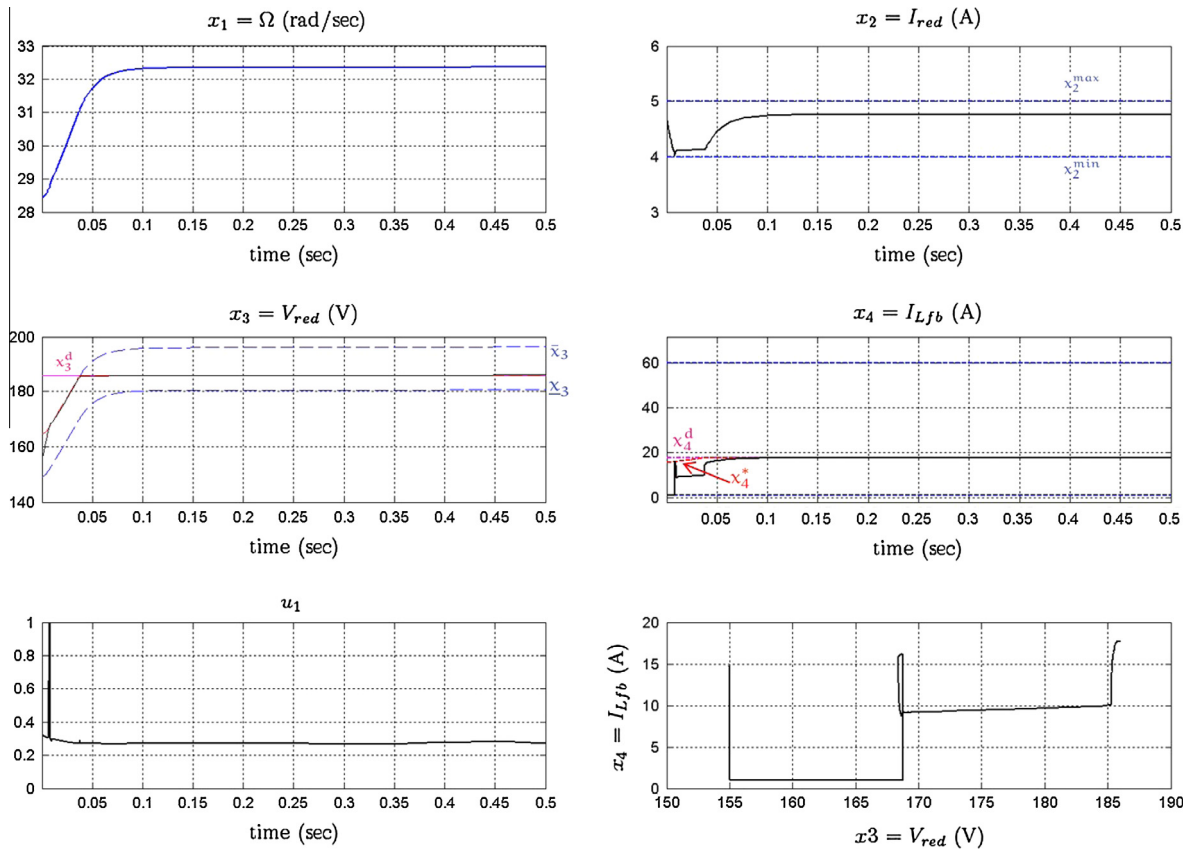


Fig. 7. Control of the (x_3, x_4) system under constant $x_5 = x_5^d$ after a step change of +20% on the desired value x_4^d . Note that the bounds $\bar{x}_3$ and $\bar{x}_3$ slow down the response in order to meet the constraints on x_2 . The same scenario is depicted in Fig. 10 using logarithmic scale in order to show the evolution of the variables during first milliseconds of the scenario.

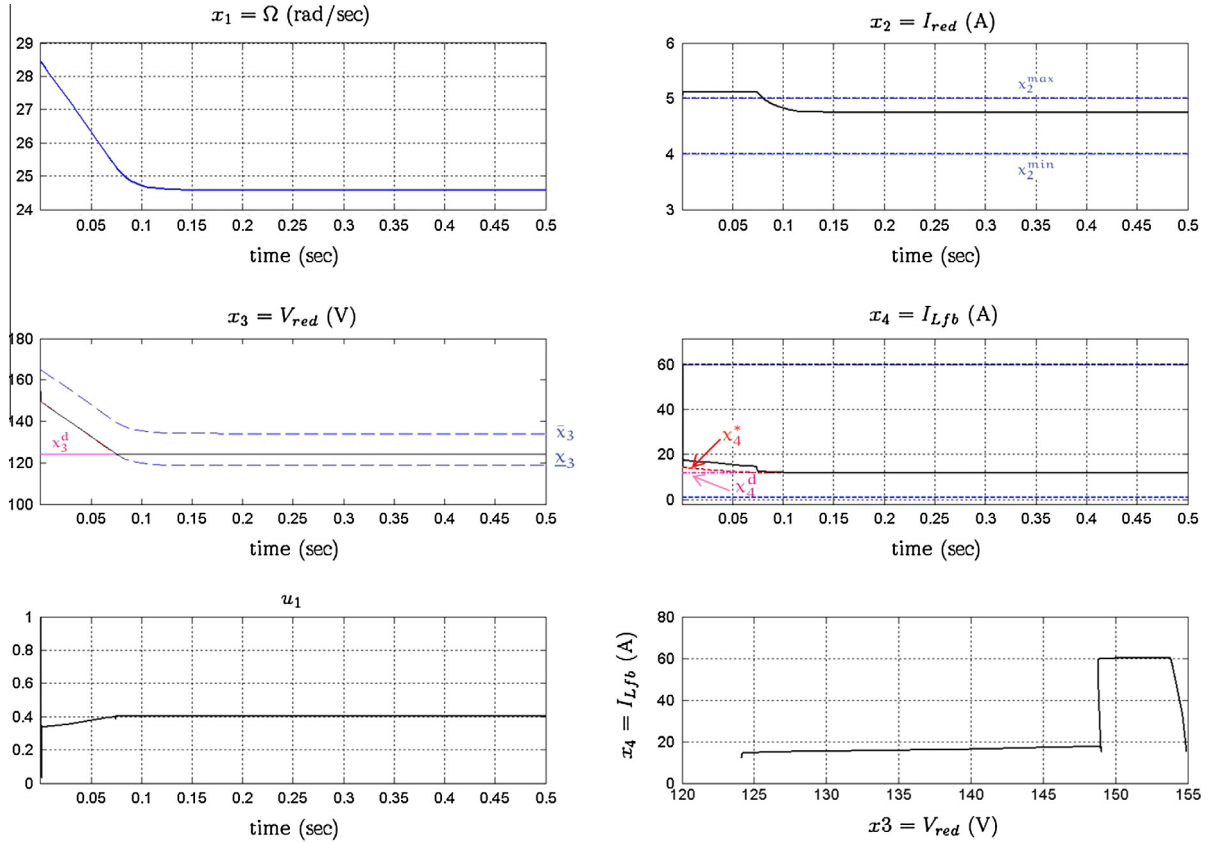


Fig. 8. Control of the (x_3, x_4) system under constant $x_5 = x_5^d$ after a step change of -20% on the desired value x_4^d . Note that the bounds $\bar{x}_3$ and $\bar{x}_4$ slow down the response in order to meet the constraints on x_2 . The same scenario is depicted in Fig. 11 using logarithmic scale in order to show the evolution of the variables during first milliseconds of the scenario.

power generation system. The control architecture explicitly handles saturations on the control variables as well as on the components of the state vector. The resulting closed-loop behavior can be used in the design step where the choice of the components and the class of power demand profiles have to be investigated. This is because such choices highly impact the size of the supercapacitor that can afford the different power demand profiles.

Beyond the specific system that has been considered in the present paper, it is our claim that the control design methodology can be extended to a wide variety of power conditioning stages that can be encountered even when other thermodynamic cycles and/or different storing devices are used.

Appendix A. Proof of Lemma 1

Let us denote by z_f and z_s the fast and the slow eigen-modes of the linear system (2a), (2b) and denote by $y = x_2$ the output we are interested in. By definition of the eigen-modes, the following expression is derived:

$$y(t) = \alpha_f z_f(t) + \alpha_s z_s(t)$$

Suppose that we have some current initial state that satisfies the saturation constraints (3) on x_2 . Let us characterize all constant profiles on x_3 leading to trajectories that still meet these constraints. According to Assumption 1 on λ_f and λ_s , the temporal generic structure of the trajectory $y(\cdot)$ is approximately the one depicted in Fig. 16 (The first order behaviors of the eigen-modes are replaced by straight lines). Namely, the position in time of the singular point (zero time-derivative) is close to the settling time (t^*) of the fastest

modes and this regardless of the value of the constant steering signal x_3 . This is because after this value t^* , the derivative of y becomes monotonic and cannot change its sign anymore.

Now for this once-for-all fixed value t^* , the expression of the output $y(t^*) = x_2(t^*)$ can be given in terms of the current value of (x_1, x_2) and the input x_3 using constant and off-line computable matrices M^* , N^* and K^* according to:

$$x_2(t^*) := M^* \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + N^* a_2 + K^* x_3 \quad (\text{A.1})$$

This obviously leads to the following expressions for $\underline{x}_3$ and $\bar{x}_3$:

$$\underline{x}_3(x_1, x_2) := \max \left\{ x_3^{\min}, \frac{1}{K^*} \left[x_2^{\max} - M^* \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - N^* a_2 \right] \right\} \quad (\text{A.2})$$

$$\bar{x}_3(x_1, x_2) := \min \left\{ x_3^{\max}, \frac{1}{K^*} \left[x_2^{\min} - M^* \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - N^* a_2 \right] \right\} \quad (\text{A.3})$$

Now if x_3 is not constant but satisfies at each instant the constraint (4), the result can be proved by obvious receding-horizon way by looking at the first order two eigen-systems. This obviously ends the proof of Lemma 1. $\square$

Appendix B. Mathematical conditions to check items (5), (6) and (7) of Assumption 4

The Item (5) of Assumption 4 can be checked through the following conditions that can be checked by scalar optimization in the decision variable x_3 :

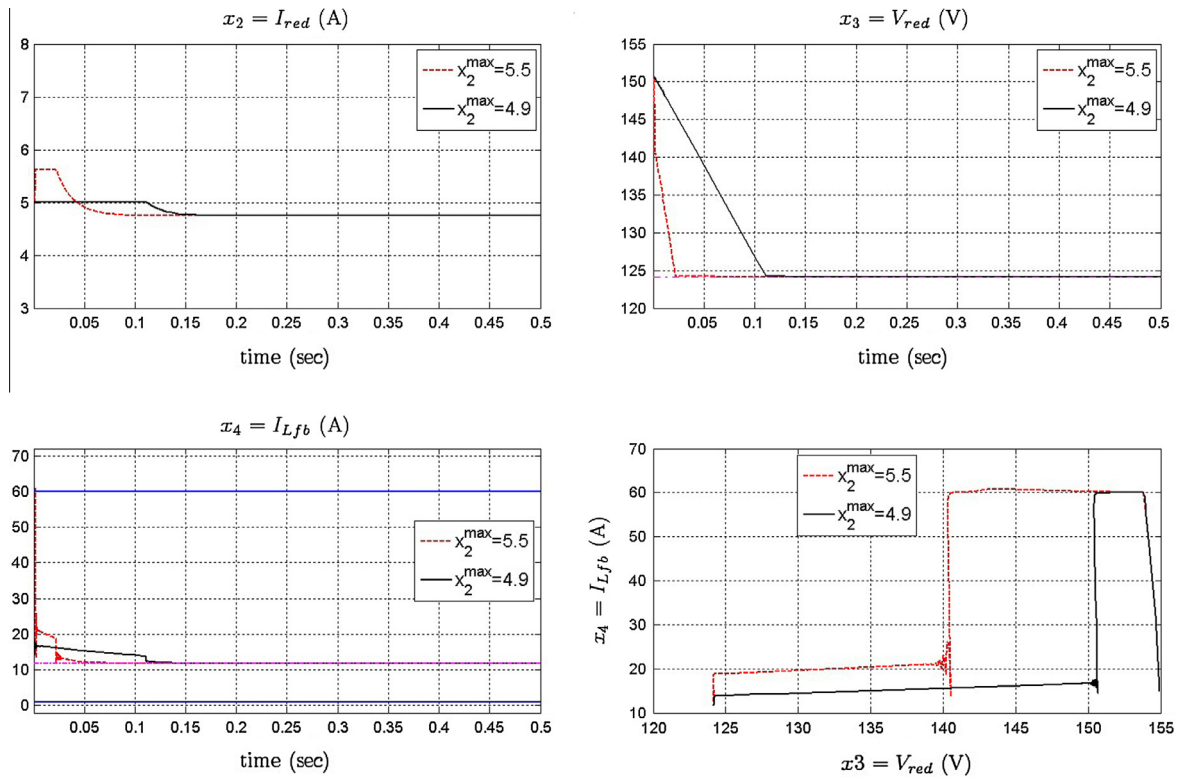


Fig. 9. Control of the (x_3, x_4) system under constant $x_5 = x_5^d$ after a step change of -20% on the desired value x_4^d . Influence of the constraints on x_2 on the response time of the system.

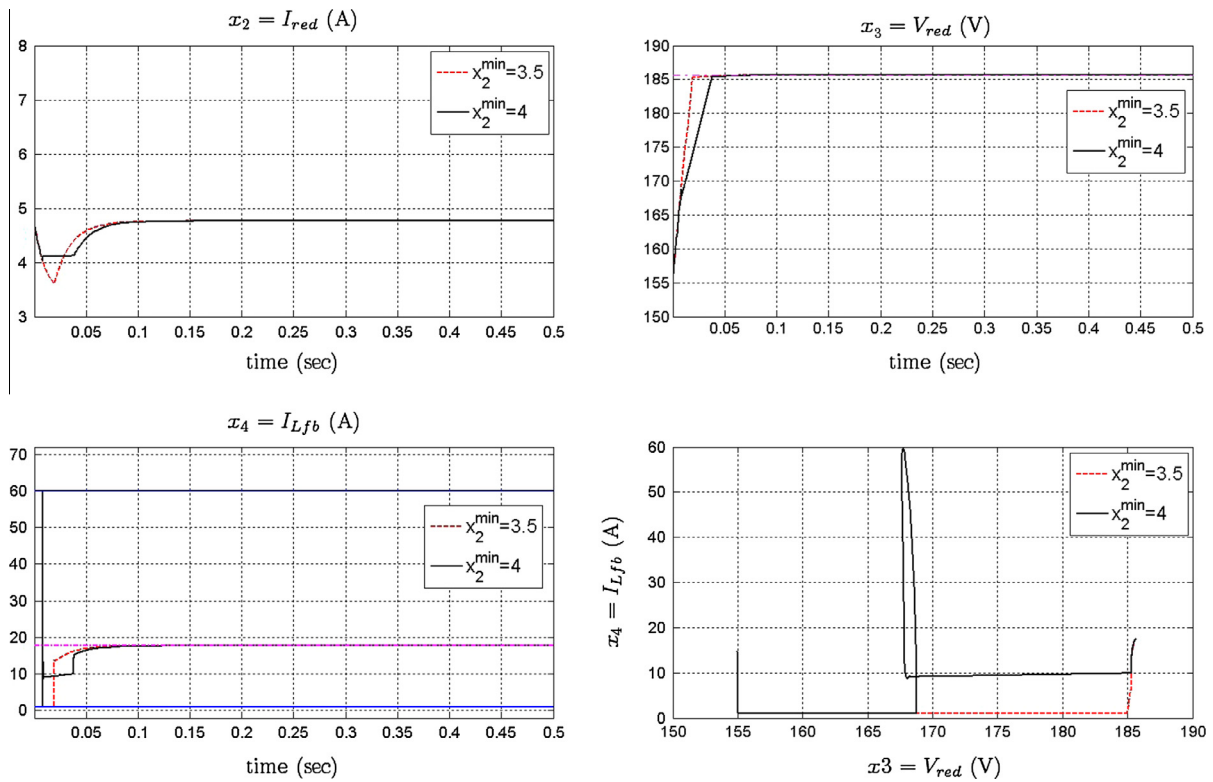


Fig. 10. Control of the (x_3, x_4) system under constant $x_5 = x_5^d$ after a step change of $+20\%$ on the desired value x_4^d . Influence of the constraints on x_2 on the response time of the system.

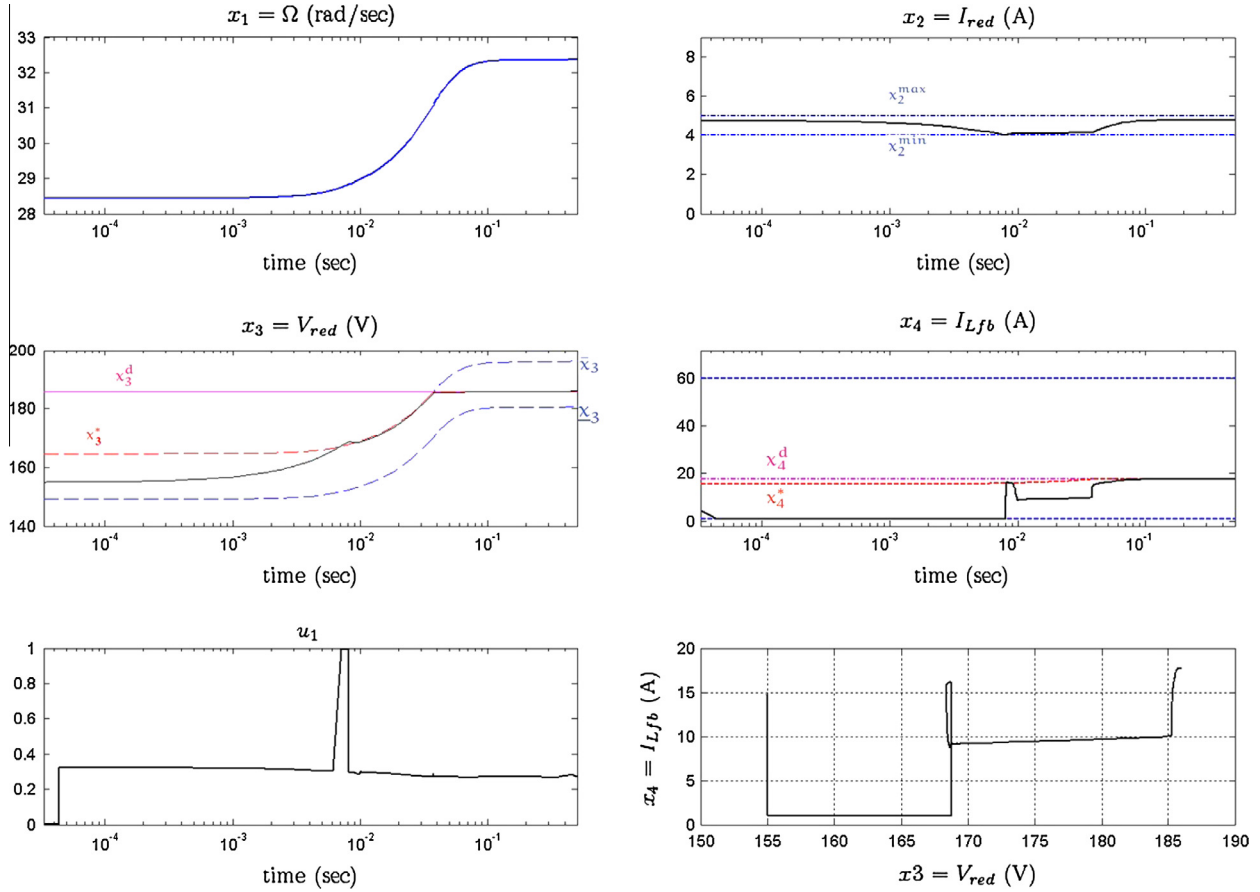


Fig. 11. Same scenario as the one depicted on Fig. 6 using logarithmic scale on the time axis in order to show the beginning of the scenario. This scale enables the saturation on x_4 to be clearly shown.

$$\min_{(x_3, x_4^{min}) \in AB} \left[x_2^{st}(x_3^*) - \frac{x_4^{min} x_5^r}{x_3} \right] > 0 \quad (B.1)$$

$$\max_{(x_3, x_4^{max}) \in DE} \left[x_2^{st}(x_3^*) - \frac{x_4^{max} x_5^r}{x_3} \right] < 0 \quad (B.2)$$

since the analyzed expression is nothing but the resulting derivative of x_3 when x_2, x_4 and x_5 take the appropriate values.

As for the items (6) and (7) regarding the attractivity of the set BCD, it can be checked through the following conditions (in which the notation $p = (x_3, x_4)^T$ is used for compactness):

$$\min_{(t, p^0) \in [0, t_f] \times \mathcal{R}_{34}} d(p(t, p^0, u_1 = 1), ABCDE) = 0 \quad (B.3)$$

$$\min_{(t, p^0) \in [0, t_f] \times \mathcal{L}_{34}} d(p(t, p^0, u_1 = 0), ABCDE) = 0 \quad (B.4)$$

where the subsets $\mathcal{R} \in \mathbb{R}^2$ and $\mathcal{L} \in \mathbb{R}^2$ are the sets that denote the values of (x_3, x_4) that lie to the right and to the left of BCD respectively. $p(t, p^0, u_1 = 1)$ denotes the trajectories of (2c)–(2d) starting from p^0 under the control $u_1 = 1$ and when $x_5 = x_5^r$ and $x_2 = x_2^{st}(x_3^*)$ are used. Similar definitions hold for $p(t, p^0, u_1 = 0)$ except that the control $u_1 = 0$ is used. The notation $d(p, ABCDE)$ denotes the distance between p and the curves ABCDE. Finally, the final time t_f is some sufficiently long computation time in order for the trajectory to cross the curve ABCDE.

Appendix C. Proof of Lemma 3

To locally regulate $x_5 = V_{bus}$ around x_5^r , A backstepping design method is applied to Eqs. 2e, 2f.

Step 1: We use the variable x_6 as a virtual control to regulate $x_5 = V_{bus}$ around x_5^r .

The regulation error being $\epsilon_5 = x_5 - x_5^r$, we use the following Lyapunov function: $V_5 = \frac{1}{2} \cdot \epsilon_5^2$. Then:

$$\dot{V}_5 = \epsilon_5 \cdot \dot{\epsilon}_5 = \epsilon_5 \cdot (a_{10} \cdot (x_4 + x_6) - \frac{a_{10}}{\eta_{inv}} \cdot \frac{P_L}{x_5})$$

By setting $\dot{V}_5 = -\rho_5 \cdot \epsilon_5^2 < 0$, with $\rho_5 > 0$ as a design parameter, we get:

$$x_6^{ref} = \frac{P_L}{\eta_{ond} \cdot x_5} - x_4 + \frac{\rho_5}{a_{10}} \cdot (x_5^r - x_5)$$

Step 2: To regulate x_6 around x_6^{ref} , the control variable u_2 is used.

The regulation error being $\epsilon_6 = x_6 - x_6^{ref}$, we use the following Lyapunov function: $V_6 = V_5 + \frac{1}{2} \cdot \epsilon_6^2$. Then:

$$\begin{aligned} \dot{V}_6 &= \epsilon_5 \cdot \dot{\epsilon}_5 + \epsilon_6 \cdot \dot{\epsilon}_6 = \epsilon_5 \cdot (a_{10} \cdot x_4 + a_{10} \cdot (x_6^{ref} + \epsilon_6) - \frac{a_{10}}{\eta_{inv}} \cdot \frac{P_L}{x_5}) \\ &\quad + \epsilon_6 \cdot (-a_{11} \cdot x_5 + a_{11} \cdot x_7 \cdot u_2 - \dot{x}_6^{ref}) \\ &= -\rho_5 \cdot \epsilon_5^2 + \epsilon_6 \cdot (-a_{11} \cdot x_5 + a_{11} \cdot x_7 \cdot u_2 - \dot{x}_6^{ref} + a_{10} \cdot \epsilon_5) \end{aligned}$$

By setting $\dot{V}_6 = -\rho_5 \cdot \epsilon_5^2 - \rho_6 \cdot \epsilon_6^2 < 0$, with ρ_6 as another design parameter, we get:

$$u_2^*(x) := \frac{1}{a_{11} \cdot x_7} \cdot (a_{11} \cdot x_5 - a_{10} \cdot (x_5 - x_5^r) + \dot{x}_6^{ref} - \rho_6 \cdot (x_6 - x_6^{ref}))$$

Appendix D. Proof of Proposition 1

Note that by definition of the interpolated feedback laws $K_1(x, P_L)$ and $K_2(x, P_L)$, the resulting closed-loop trajectories passes

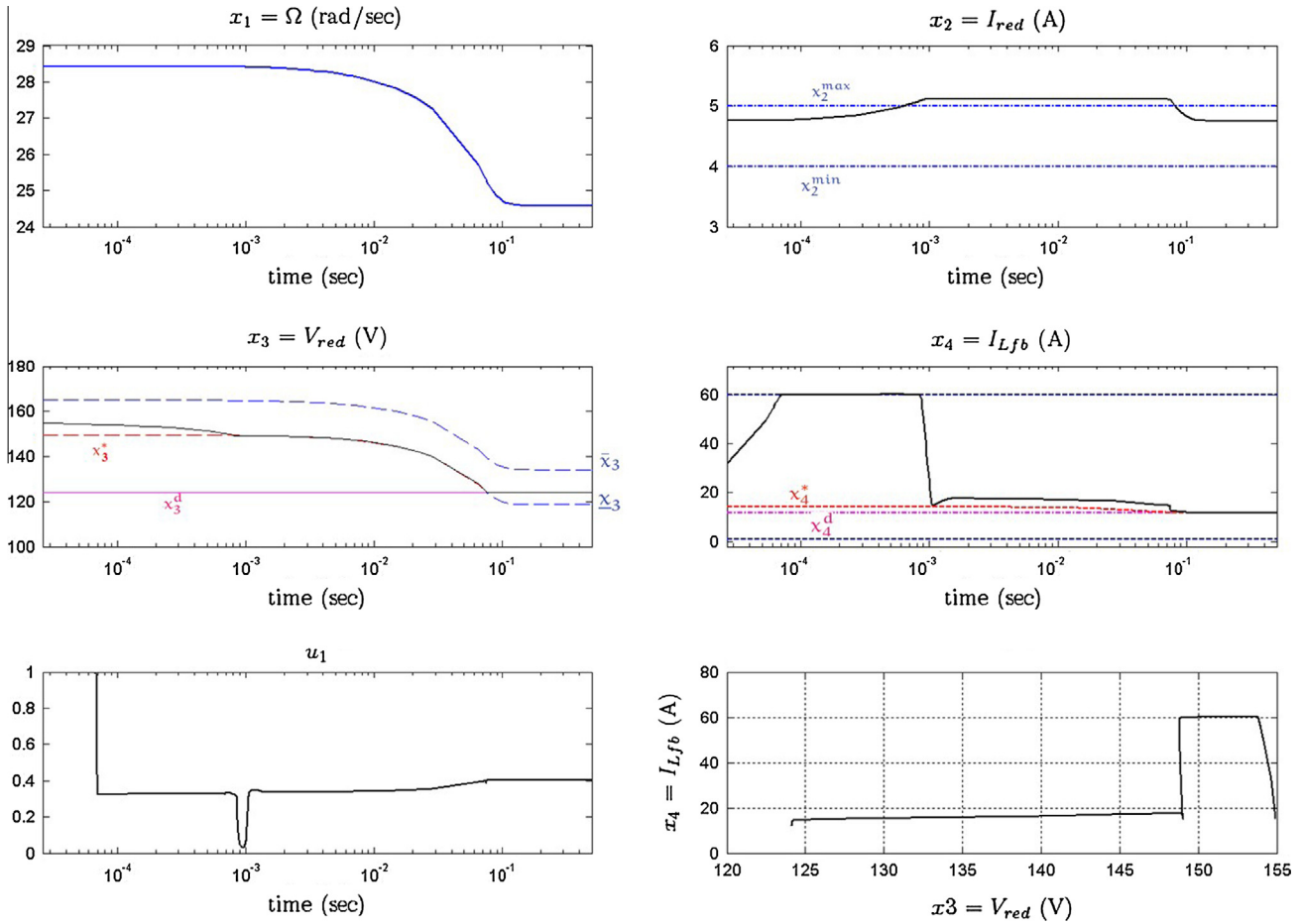


Fig. 12. Same scenario as the one depicted on Fig. 7 using logarithmic scale on the time axis in order to show the beginning of the scenario. This scale enables the saturation on x_4 to be clearly shown.

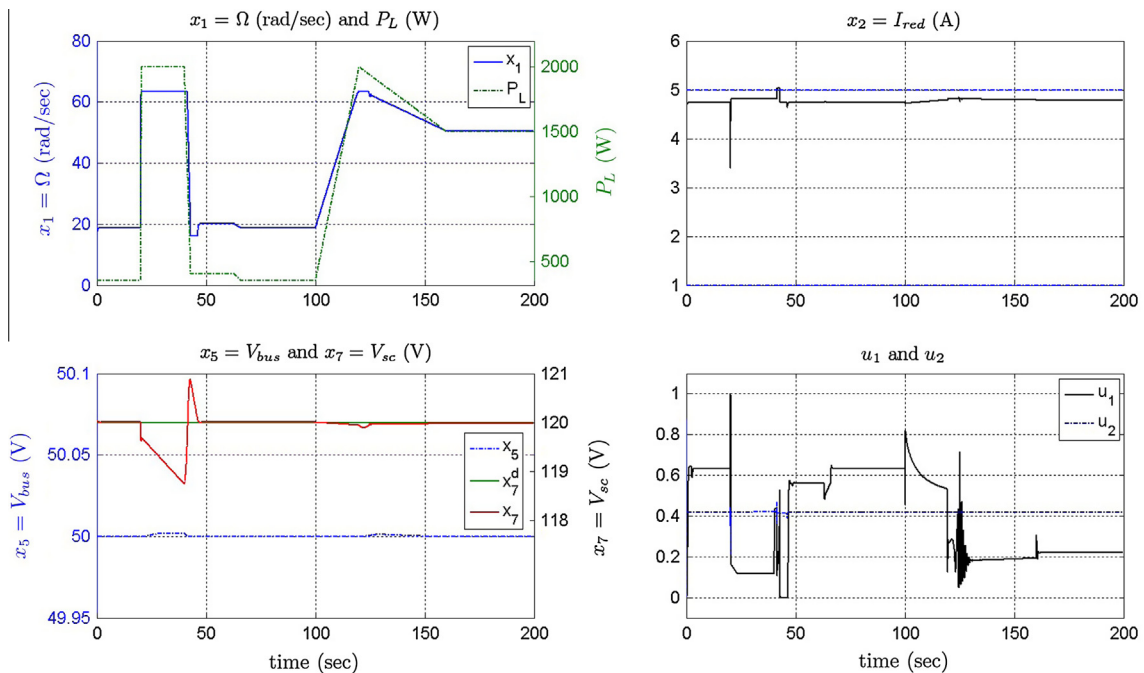


Fig. 13. Performance of the overall closed-loop system under varying power load demand. Case where $x_2^{max} = 5$. This scenario is worth comparing with the scenario depicted in Fig. 13 where the upper bound $x_2^{max} = 4.8$ is used.

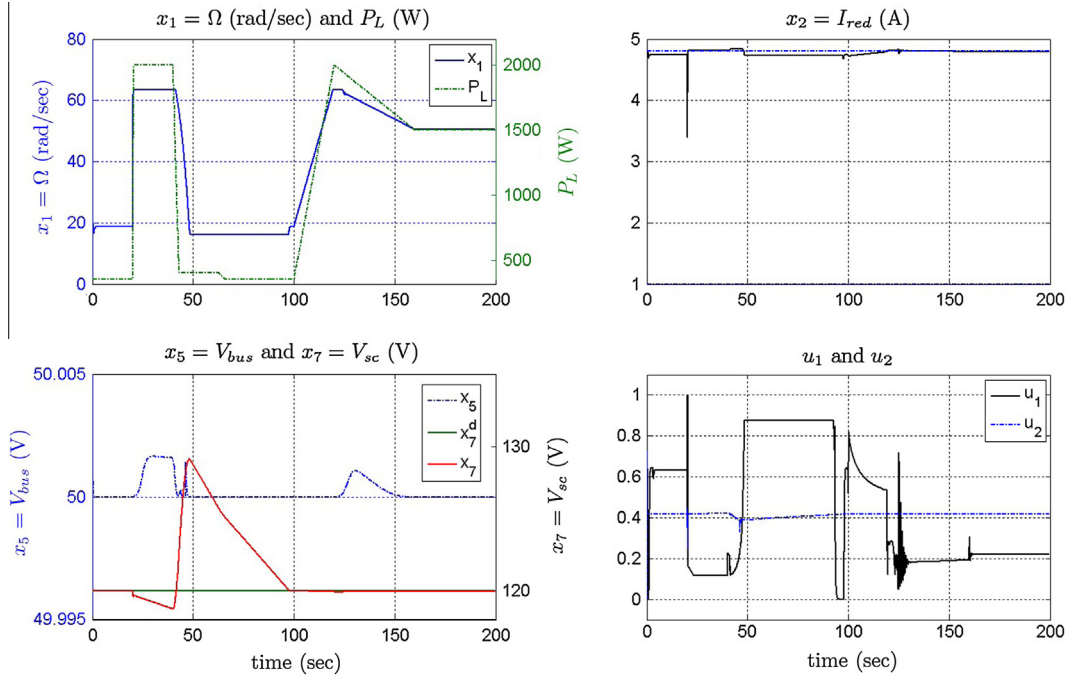


Fig. 14. Performance of the overall closed-loop system under varying power load demand. Case where $x_2^{max} = 4.8$. This scenario is worth comparing with the scenario depicted in Fig. 12 where the upper bound $x_2^{max} = 5$ is used.

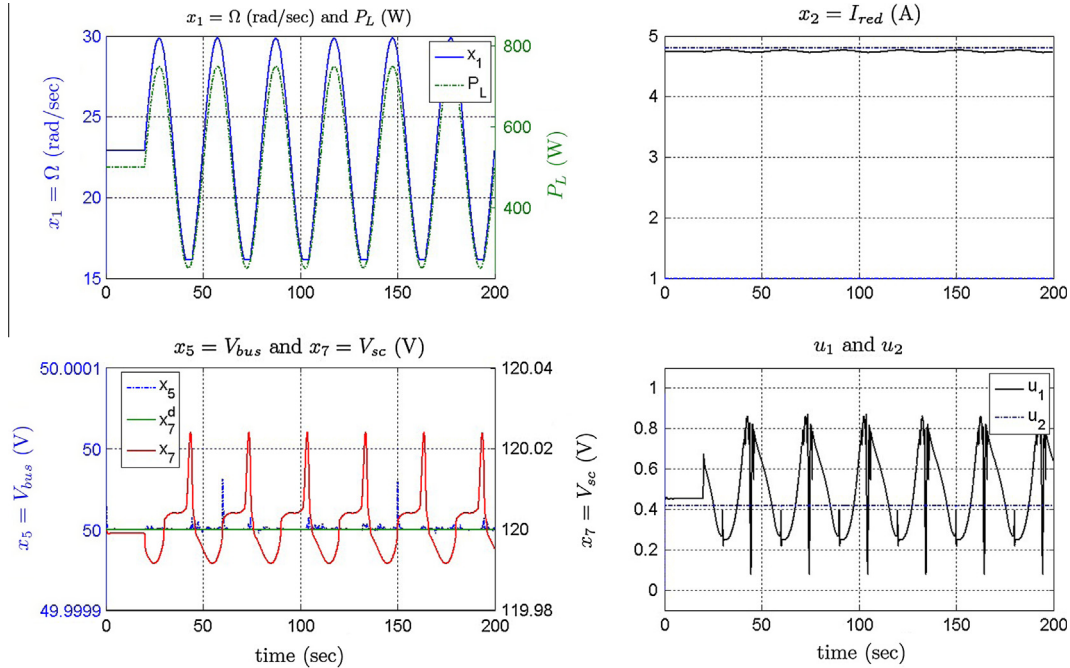


Fig. 15. Performance of the overall closed-loop system under sinusoidal power load demand.

through neighborhoods $\mathcal{V}_{34}(x_3^*, x_4^*)$ and $\mathcal{V}_{56}(x_5^r, x_6^r)$ of the targeted pairs (x_3^*, x_4^*) and (x_5^r, x_6^r) of size $O(1/\lambda)$ after a finite time.

Let first $\gamma_{34} > 0$ and $\gamma_{56} > 0$ be taken sufficiently small to guarantee that the corresponding level sets belong to the regions of attraction of the local controls $u_1^t(x_3(x), P_L)$ and $u_2^t(x, P_L)$. Once this is done, take $\lambda > 0$ sufficiently high to guarantee that each of the conditions $d_{34}(x, P_L) < \gamma_{34}$ and $d_{56}(x, P_L) < \gamma_{56}$ will be reached in finite time so that the local controllers can be fired.

Now since the switching conditions in (28), (29) are defined using the Lyapunov level sets associated to the local stabilizing

control laws, no chattering behavior occurs and the state is asymptotically steered to the stationary state x^{st} defined by:

$$x_i^{st}(x_4^d) \quad i = 1, \dots, 4 \quad (D.1)$$

$$x_5^{st} = x_5^r \quad (D.2)$$

$$x_6^{st} = 0 \quad (D.3)$$

$$x_7^{st} = x_7^r \quad (D.4)$$

where x_4^d is given by:

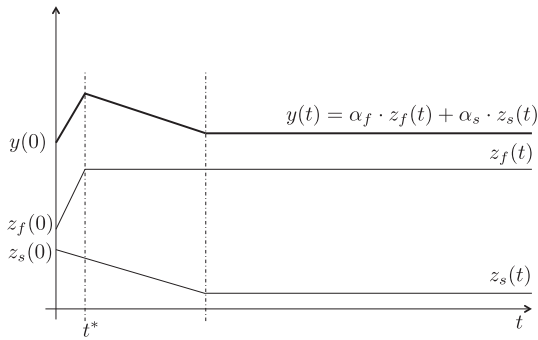


Fig. 16. Illustration of the proof of Lemma 1: Under the conditions of the Lemma, the position of the pic of $y = x_2$ is almost independent of x_3 .

$$x_4^d := \frac{P_L}{\eta_{inv} x_5^r} \quad (\text{D.5})$$

Indeed, this last equation comes from (14) because of the asymptotic convergence of x_7 towards x_7^r that results from (15). This with (13) shows that x_6 converges to 0. This obviously ends the proof. $\square$

References

- [1] Kalantar M, Mousavi G. Dynamic behavior of stand-alone hybrid power generation system of wind turbine, solar array and battery storage. *Appl Energy* 2010;87:3051–64.
- [2] Rahmani MA, Alamir M, Gualino D. Control strategy for an off-grid Stirling engine/supercapacitor power generation system. In: Proceedings of the American control conference, Washington, DC, USA; 2013.
- [3] Haque ME, Negnevitsky M, Muttaqui KM. A novel control strategy for a variable speed wind turbine with a permanent magnet synchronous generator. *IEEE Trans Indus Appl* 2010;46(1):331–9.
- [4] Mendis N, Muttaqui KM, Perera S, Uddin MN. A novel control strategy for stand-alone operation of a wind dominated RAPS system. *Industry applications society annual meeting (IAS)*; 2011. p. 1–8.
- [5] Haruni AMO, Gargoom A, Haque ME, Negnevitsky M. Dynamic operation and control of a hybrid wind-diesel stand alone power systems. In: 2010 Twenty-fifth annual IEEE applied power electronics conference and exposition (APEC); 2010. p. 162–9.
- [6] Ahmed NA, Al-Othman AK, Alrashidi MR. Development of an efficient utility interactive combined wind/photovoltaic/fuel cell power system with MPPT and DC bus voltage regulation. *Electr Power Syst Res* 2011;81:1096–106.
- [7] Valenciaga F, Puleston PF, Battaiotto PE, Mantz RJ. Passivity/sliding mode control of a stand-alone hybrid generation system. *IEE Proc-Control Theory Appl* 2000;147(6):680–6.
- [8] Valenciaga F, Puleston PF, Battaiotto PE. Variable structure system control design method based on a differential geometric approach: application to a wind energy conversion subsystem. *IEE Proc-Control Theory Appl* 2004;151(1):6–12.
- [9] Qi W, Liu J, Chen X, Christofides PD. Supervisory predictive control of standalone wind/solar energy generation systems. *IEEE Trans Control Syst Technol* 2011;19(1):199–207.
- [10] Valenciaga F, Puleston PF. Supervisor control for a stand-alone hybrid generation system using wind and photovoltaic energy. *IEEE Trans Energy Convers* 2005;20(2):398–405.
- [11] Valenciaga F, Puleston PF. High-order sliding control for a wind energy conversion system based on a permanent magnet synchronous generator. *IEEE Trans Energy Convers* 2008;23(3):860–7.
- [12] Qi W, Liu J, Christofides PD. A two-time-scale framework to supervisory predictive control of an integrated wind/solar energy generation and water desalination system. In: 2011 American control conference; 2011. p. 2677–82.
- [13] Mathieu A. Contribution à la conception et à l'optimisation thermodynamique d'une micro-centrale solaire thermodynamique. PhD dissertation, Université de Lorraine, Nancy; 2012.
- [14] Meyer D. Modeling and control of a stirling engine for a solar micro plant. Master thesis dissertation, control systems department, University of Grenoble; 2011.
- [15] Schmidt M, Lipson H. Distilling free-form natural laws from experimental data. *Science* 2009;324(5923):81–5.
- [16] Schmidt M, Lipson H. Eureka (Version 0.98 beta) [Software]. <<http://www.eureka.com/>>.